9. PID Controller

EN2142 Electronic Control Systems

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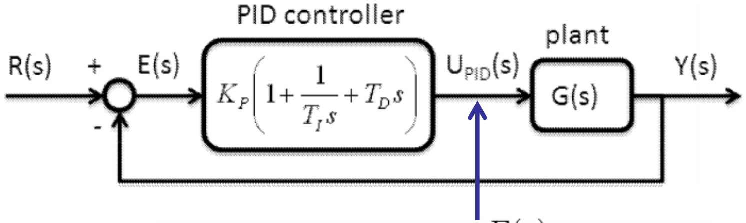
Three Term Controller

- Most popular industrial controller
- Can handle any industrial plant easily and satisfactorily
- Three gain controller $[K_P \; K_D \; K_I]$ or $[K_P \; T_D \; T_I]$
- **P**roportional, **D**erivative, and **I**ntegral control actions
- Weighted sum of three terms using gains



$$U_{PID}(s) = K_P e(s) + K_I \frac{E(s)}{s} + K_D s E(s)$$

$$U_{PID}(s) = K_P \left\{ E(s) + \frac{1}{T_I}\frac{E(s)}{s} + T_D s E(s) \right\}$$

$$T_i = \frac{K_P}{K_I}$$

$$T_d = \frac{K_D}{K_P}$$

Proportional Term

- Determines the **responsiveness** of the controller
  - Aggressive/Weak
  - Increase/Decrease system bandwidth

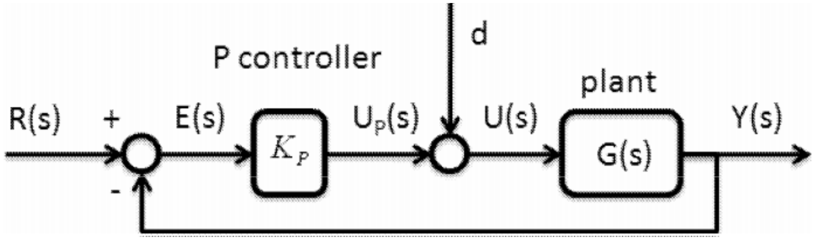
$$e(t) = r(t) - y(t)$$

- Positive command when $r(t) > y(t)$
- Negative command when $r(t) < y(t)$

- K_P is very BIG
  - Controller is sensitive
  - Responds to even small errors)
- K_P is very small
  - Controller is not sensitive
  - Respond only to BIG errors

Both extremes are not acceptable

Error due to Persistent Disturbances



- At equilibrium state

$$K_P e_{ss} + d = 0$$

$$e_{ss} = -\frac{d}{K_P}$$

P controller alone cannot eliminate the steady state error

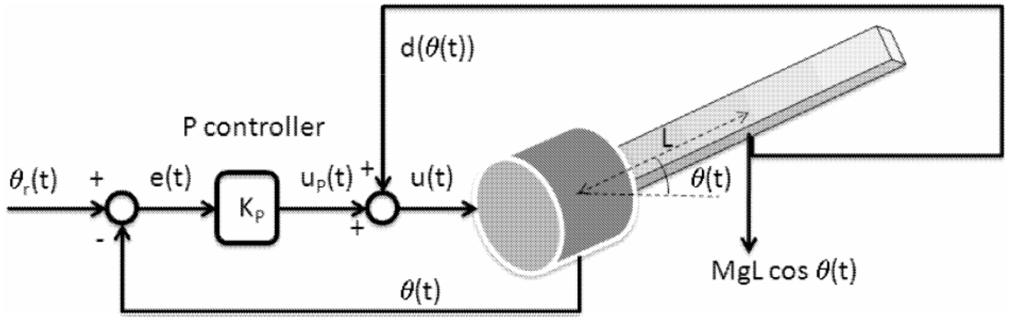
- Introduce an I-controller. Then, at equilibrium state

$$K_p e(t) + K_I \int e(t)dt + d = 0$$

$$e(t) = 0 \text{ and } \int e(t)dt = -d$$

Example

- Robot link position P-control system $K_p = 25,\ M = 3\text{kg}$ $L = 1\text{m},\ g = 9.8\text{ms}^{-2}$

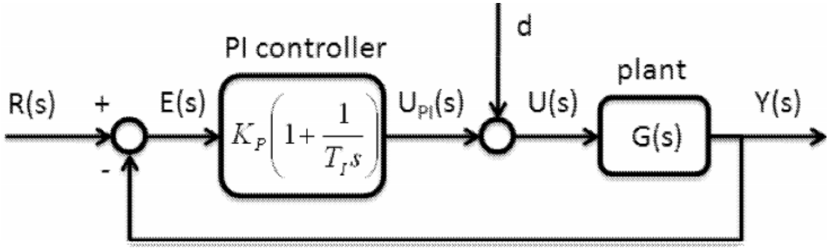


P controller needs an error to generate torque to counter gravity, then reach steady state

$$\begin{aligned} K_p(\theta_r - \theta_{ss}) - MgL\cos\theta_{ss} &= 0 \\ \theta_{ss} + \frac{MgL\cos\theta_{ss}}{K_p} - \theta_r &= 0 \\ \theta_{ss} + \frac{3\times 9.8\times 1\cos\theta_{ss}}{25} - 1 &= 0 \\ \theta_{ss} = 0.15\text{rad} \longleftarrow \theta_{ss} + 0.85\cos\theta_{ss} - 1 &= 0 \end{aligned}$$

I Controller

- Can eliminate steady state error
- If $y(t) < r(t)$ then $e(t) > 0$ $u_I(t) = \int e(t)dt > 0$ drives $y(t) \to r(t)$



- Caution: Error accumulation together, with a BIG value of K_I could generate large control commands causing response $y(t)$ to overshoot/undershoot (stability problem)
  - Reduce $K_I (= K_P / T_I)$ to reduce overshoot but it will take time to correct steady state errors

D - Controller

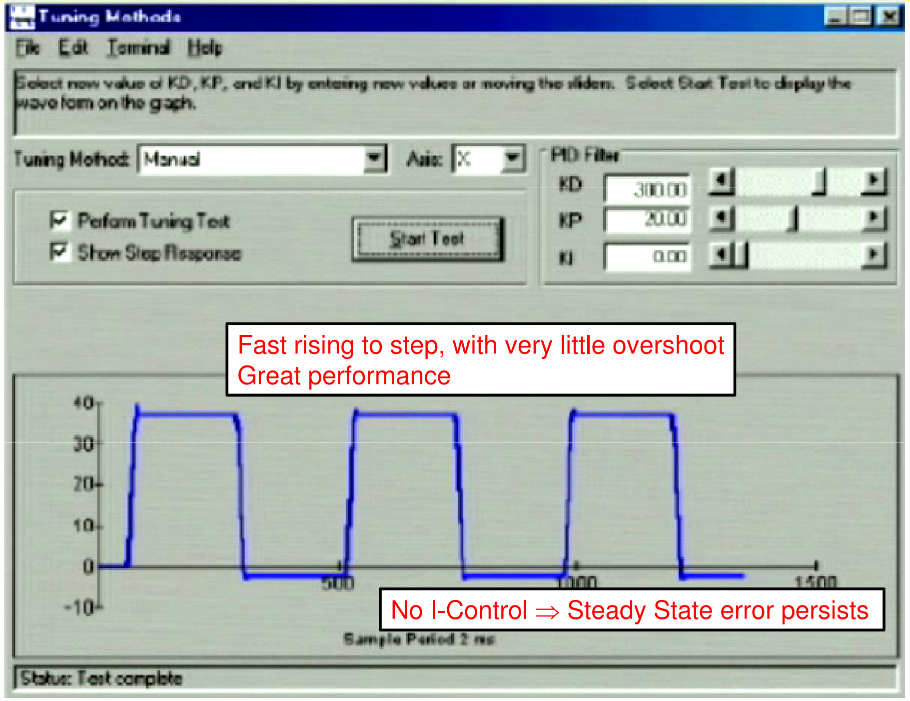
- Corrective action is proportional to the rate of change of error $\dot{e}(t) = \dot{r}(t) - \dot{y}(t)$
- For constant reference $\dot{r}(t) = 0$ and $u_D(t) = -K_D\dot{y}(t)$

- If a **negative error is dropping faster** (**unstable?**), strong positive action is taken to stop and correct it
- If a **positive error is rising faster** (**unstable?**), strong negative action is taken

Improves Stability

- When the error doesn't change, no control action is taken (doesn't try to correct steady errors)

Tuning PD Gains in Motion Control



Select new value of KD, KP, and KI by entering new values or moving the sliders. Select Start Test to display the wave form on the graph.

Tuning Method: Manual
Axis: X

PID Filter
KD 300.00
KP 20.00
KI 0.00

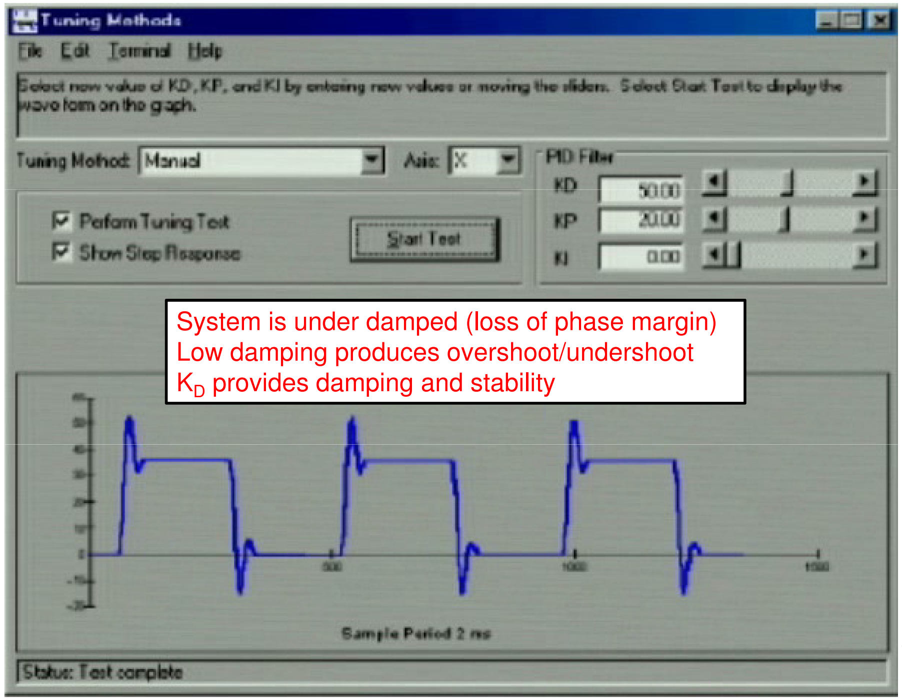
☑ Perform Tuning Test
☑ Show Step Response

Fast rising to step, with very little overshoot
Great performance

No I-Control ⇒ Steady State error persists

Status: Test complete

When K_D is too Low



Select new value of KD, KP, and KI by entering new values or moving the sliders. Select Start Test to display the wave form on the graph.

Tuning Method: Manual

Axis: X

KD 50.00

KP 20.00

KI 0.00

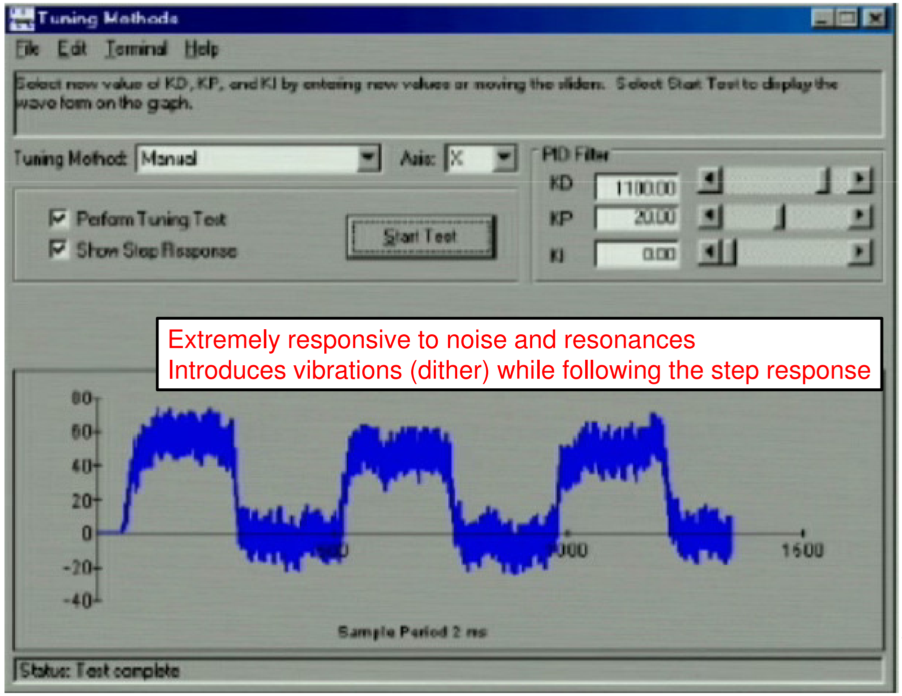
☑ Perform Tuning Test

☑ Show Step Response

Status: Test complete

System is under damped (loss of phase margin)
Low damping produces overshoot/undershoot
K_D provides damping and stability

When K_D is too High



Select new value of KD, KP, and KI by entering new values or moving the sliders. Select Start Test to display the wave form on the graph.

Tuning Method: Manual

Axis: X

KD 1100.00

KP 20.00

KI 0.00

☑ Perform Tuning Test

☑ Show Step Response

Status: Test complete

Extremely responsive to noise and resonances
Introduces vibrations (dither) while following the step response

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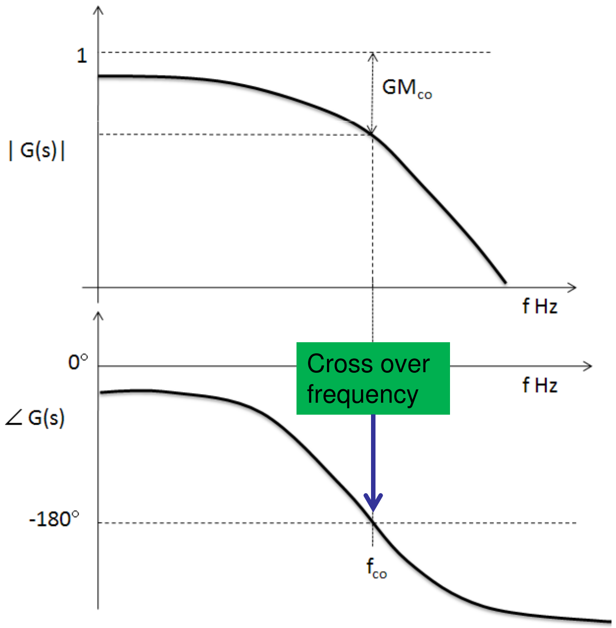
Tuning PID Controller

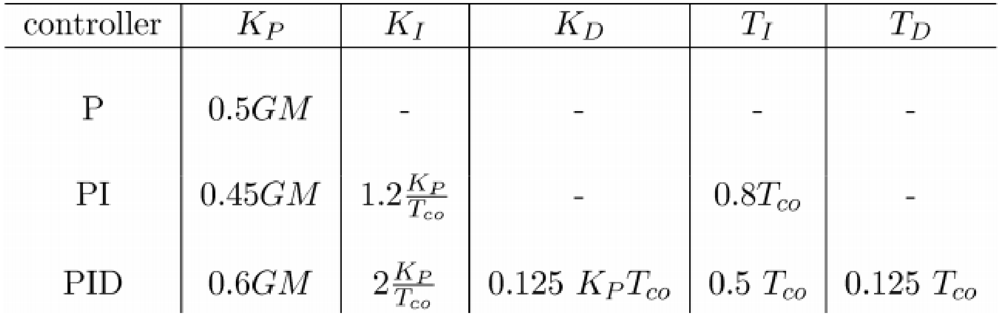
- Best match between $[K_P\ K_D\ K_I]$ or $[K_P\ T_D\ T_I]$
- K_P makes the system responsive to errors, however, a bigger value of K_P will make the system too sensitive, and responsive to even noise in the control loop. K_I reduces the steady state error, however, it increases overshoot and reduces stability. K_D stabilizes the system by slowing down the response.
- In order to realize desirable response the three individual controllers have to be properly adjusted
- There are three main techniques for PID controller tuning
  - Zeigler-Nichols
  - Cohen-Coon
  - ITAE based methods

Ziegler-Nochols Method

- Frequency response of the plant is required
- Plant needs to have a crossover frequency f_{co} and stable gain margin Gm_{co}



Ziegler-Nochols Method



controller	K_P	K_I	K_D	T_I	T_D
P	$0.5GM$	-	-	-	-
PI	$0.45GM$	$1.2\frac{K_P}{T_{co}}$	-	$0.8T_{co}$	-
PID	$0.6GM$	$2\frac{K_P}{T_{co}}$	$0.125\ K_P T_{co}$	$0.5\ T_{co}$	$0.125\ T_{co}$